Instabilities and the null energy condition

Roman V. Buniy* and Stephen D.H. Hsu†
Institute of Theoretical Science, University of Oregon, Eugene OR 94703-5203

We show that violation of the null energy condition implies instability in a broad class of models, including classical gauge theories with scalar and fermionic matter as well as any perfect fluid. When applied to the dark energy, our results imply that $w = p/\rho$ is unlikely to be less than -1.

I. INTRODUCTION

Energy conditions, or restrictions on the matter energy-momentum tensor $T_{\mu\nu}$, play an important role in general relativity. No classification of the solutions to Einstein's equation is possible without restrictions on $T_{\mu\nu}$, since every spacetime is a solution for some particular choice of energy-momentum tensor. In this letter we demonstrate a direct connection between stability and the null energy condition (NEC) [1], $T_{\mu\nu}n^\mu n^\nu \geq 0$ for any null vector n (satisfying $g_{\mu\nu}n^\mu n^\nu = 0$). Our main results are: (1) classical solutions of scalar-gauge models which violate the NEC are unstable, (2) a quantum state (including fermions) in which the expectation of the energy-momentum tensor violates the NEC cannot be the ground state, (3) perfect fluids which violate the NEC are unstable. These results suggest that violations of the NEC in physically interesting cases are likely to be only ephemeral.

Our results have immediate applications to the dark energy equation of state, often given in terms of $w = p/\rho$. Dark energy has positive energy density ρ and energy-momentum tensor $T_{\mu\nu} = \mathrm{diag}\,(\rho, p, p, p)$ in the comoving cosmological frame. Therefore, $w < -1$ implies violation of the NEC. Instability as a consequence of $w < -1$ was studied previously in scalar models [2].

Some results in relativity in which the NEC plays an important role include the classical singularity theorems [3], proposed covariant entropy bounds [4] and non-existence of Lorentzian wormholes [5].

II. FIELD THEORIES

Consider a theory of scalar, ϕ_a, and gauge, $A_{a\alpha}$, fields in a fixed d-dimensional space-time with the metric $g_{\mu\nu}$. We limit ourselves to theories whose equations of motion are second order differential equations, so the Lagrangian for the system is assumed to depend only on the fields and their first derivatives. We take the Lagrangian density $\mathcal{L}$ to depend only on the covariant derivative of the field $D_\mu\phi_a$ and the gauge field strength $F_{a\mu\nu}$. The scalars may transform in any representation of the gauge group. We impose Lorentz invariance on $\mathcal{L}$, but do not require overall gauge invariance. That is, we allow for fixed tensors with gauge indices (but no Lorentz indices) which can be contracted with the fields. For the corresponding action

$$S = \int d^dx\, |g|^{\frac{1}{2}} \mathcal{L}(\phi_a, D_\mu\phi_a, F_{a\mu\nu}) \tag{1}$$

to be stationary, its first variation has to vanish, $\delta S = 0$. This leads to the equations of motion for the fields ϕ_a and $A_{a\alpha}$; in the classical analysis we assume that we have found solutions to these equations, about which we expand.

A. Null energy condition

The quantities $D_\mu\phi_a$ and $g^{\mu\nu}$ are independent variables. Nevertheless we now prove that there is a relation between the derivatives of $\mathcal{L}$ with respect to them:

$$2\mathcal{L}_{g^{\mu\nu}} = M^{AB}\psi_{A\mu}\psi_{B\nu} + g_{\mu\nu}K, \tag{2}$$

$$\mathcal{L}_{\psi_{A\mu}} = M^{AB}\psi_B{}^\mu + \epsilon^{\mu\nu_2\ldots\nu_d}L^A{}_{\nu_2\ldots\nu_d}. \tag{3}$$

In our notation $\psi_{A\mu} = (D_\mu\phi_a, F_{a\alpha\mu})$, where the abstract index A may run over both Lorentz and color indices, as well as the type of field. So, $\psi_{A\mu}$ is a list of objects, each of which has a Lorentz index μ. The value of A specifies an element of this list.

The relations are obtained by noting that for each and every $g^{\mu\nu}$ in $\mathcal{L}$ there are two ψs attached to it, except for the curved space totally antisymmetric tensor $|g|^{-\frac{1}{2}}\epsilon^{\nu_1\ldots\nu_d}$, which gives rise to the K term in Eq. (2). Similarly, differentiation with respect to $\psi_{A\mu}$ yields the M and L terms in Eq. (3).

Figure 1 represents the most general Lagrangian of type considered in this paper. Each dot represents a Lorentz index and a line connecting them denotes contraction using the metric. Rectangles (with two indices) are field strengths, small circles covariant derivatives of scalar fields, and a large circle an epsilon tensor. Finally, the block X represents the remainder of the diagram. Because the product of two epsilon tensors can be rewritten as a sum of products of metric tensors g, we need to consider only figures with at most one epsilon tensor, and therefore can assume that X contains none. (For generality, we include an epsilon tensor in the figure, although of course $\mathcal{L}$ need not contain a parity-violating component.) All Lorentz indices are ultimately contracted, and we suppress color indices for simplicity.

*Electronic address: roman@uoregon.edu
†Electronic address: hsu@duende.uoregon.edu

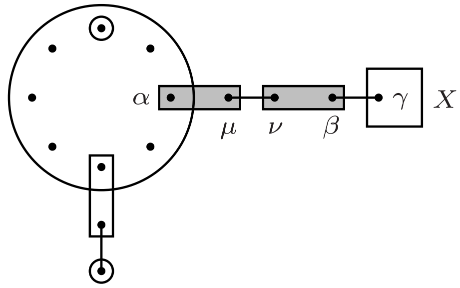

FIG. 1: Representation of the most general Lagrangian of the type considered in this paper. Each dot represents a Lorentz index, and a line connecting them denotes contraction using the metric. Rectangles (with two indices) are field strengths, small circles covariant derivatives of scalar fields, and large circles epsilon tensors. Finally, the block X represents the remainder of the diagram. All Lorentz indices are ultimately contracted, and we suppress color indices for simplicity. In graphical terms, M is obtained by simply removing the shaded elements.

Consider the labelled portion of the figure, which equals

$$\mathcal{L} = |g|^{-\frac{1}{2}} \epsilon^{\alpha\cdots} F_{\alpha\mu}\, g^{\mu\nu} F_{\nu\beta}\, g^{\beta\gamma} X_{\gamma\ldots}. \tag{4}$$

Again, we suppress color indices for simplicity, as they do not affect the proof. We include an epsilon tensor in the analysis, although $\mathcal{L}$ may or may not contain one (in the parity-preserving case the epsilon tensor in Eq. (4) is replaced by the metric). By differentiation of the indicated portion, we obtain Eqs. (2) and (3) with

$$\begin{aligned} M^{\alpha\beta} &= -|g|^{-\frac{1}{2}} \left(\epsilon^{\alpha\cdots} g^{\beta\gamma} + \epsilon^{\beta\cdots} g^{\alpha\gamma}\right) X_{\gamma\ldots}, && (5)\\ K &= |g|^{-\frac{1}{2}} \epsilon^{\alpha\cdots} F_{\alpha\rho}\, g^{\rho\sigma} F_{\sigma\beta}\, g^{\beta\gamma} X_{\gamma\ldots}, && (6)\\ L^{\alpha}{}_{\ldots} &= -|g|^{-\frac{1}{2}} g^{\alpha\rho} F_{\rho\beta}\, g^{\beta\gamma} X_{\gamma\ldots}. && (7)\end{aligned}$$

Note the derivative $\mathcal{L}_{F_{\nu\beta}}$ generates a contribution to M which is matched by a corresponding contribution from $2\mathcal{L}_{g^{\beta\gamma}}$. Other contractions of fields with $g_{\mu\nu}$ (i.e., as indicated in the figure) can be analyzed similarly. In graphical terms, M can be obtained from the figure for $\mathcal{L}$ by simply removing two ψs, in this case the shaded elements of the figure.

For the energy-momentum tensor which couples to gravity,

$$T_{\mu\nu} = -\mathcal{L} g_{\mu\nu} + 2\mathcal{L}_{g^{\mu\nu}}, \tag{8}$$

the NEC then requires

$$\Psi_A M^{AB} \Psi_B \geq 0, \tag{9}$$

where $\Psi_A = \psi_{A\mu} n^{\mu}$. Thus, to satisfy the NEC, M^{AB} has to be positive semidefinite. This property is crucial for stability of solutions, to which we now turn.

B. Stability

To study the stability of the solution $\psi_A(x)$, we consider the second variation of the Lagrangian,

$$\begin{aligned} \delta^2\mathcal{L} &= \mathcal{L}_{\psi_A\psi_B}\delta\psi_A\delta\psi_B + 2\mathcal{L}_{\psi_A\psi_{B;\lambda}}\delta\psi_A\delta\psi_{B;\lambda} \\ &+ \mathcal{L}_{\psi_{A;\mu}\psi_{B;\nu}}\delta\psi_{A;\mu}\delta\psi_{B;\nu}. \end{aligned} \tag{10}$$

Here quantities $\mathcal{L}_{\psi_A} = \partial\mathcal{L}/\partial\psi_A$, etc. are evaluated at $\psi_A(x)$. Also notice that $\psi_{A;\mu} = (D_\mu\phi_a, A_{a\alpha;\mu})$, the covariant derivatives of ψ_A, are different from $\psi_{A\mu} = (D_\mu\phi_a, F_{a\alpha\mu})$.

Let us use a locally inertial frame in which the metric is reduced to $\bar{g}_{\mu\nu} = \mathrm{diag}\,(1, -1, \ldots, -1)$; all quantities in this frame are designated by a bar. For the Lagrangian (10), the canonical momentum is

$$\delta\bar{\pi}^A = 2\mathcal{L}_{\bar{\psi}_B\bar{\psi}_{A;0}}\delta\bar{\psi}_B + 2\mathcal{L}_{\bar{\psi}_{A;0}\bar{\psi}_{B;\nu}}\delta\bar{\psi}_{B;\nu} \tag{11}$$

which leads to the following effective Hamiltonian for fluctuations about the classical solution:

$$\begin{aligned} \delta^2\mathcal{H} &= -\mathcal{L}_{\bar{\psi}_A\bar{\psi}_B}\delta\bar{\psi}_A\delta\bar{\psi}_B - 2\mathcal{L}_{\bar{\psi}_A\bar{\psi}_{B;j}}\delta\bar{\psi}_A\delta\bar{\psi}_{B;j} \\ &+ \mathcal{L}_{\bar{\psi}_{A;0}\bar{\psi}_{B;0}}\delta\bar{\psi}_{A;0}\delta\bar{\psi}_{B;0} - \mathcal{L}_{\bar{\psi}_{A;i}\bar{\psi}_{B;j}}\delta\bar{\psi}_{A;i}\delta\bar{\psi}_{B;j}. \end{aligned} \tag{12}$$

Here $\delta\bar{\psi}_{A;0}$ are functions of $\delta\bar{\pi}^B$, $\delta\bar{\psi}_B$ and $\delta\bar{\psi}_{B;i}$ as found from Eq. (11). The first term on the right hand side of Eq. (12) is a potential term, which we denote by $\delta^2\mathcal{V}$, and the last two terms are kinetic terms, denoted $\delta^2\mathcal{K}$.

If the kinetic energy $\delta^2\mathcal{K}$ is negative, then the system described by the Hamiltonian $\delta^2\mathcal{H} = \delta^2\mathcal{K} + \delta^2\mathcal{V}$ is (locally) unstable. If $\delta^2\mathcal{V}$ is positive then small perturbations will cause the classical solutions to grow exponentially away from the original stationary point. However, it is possible to have classical stability if one chooses $\delta^2\mathcal{V}$ to be negative; in this case we have an upside-down potential with negative kinetic term, or a "phantom". Such models necessarily exhibit quantum instabilities [6]. Notice, the second term in Eq. (12) is linear in the fluctuations and their derivatives, and therefore can never stabilize the system.

To investigate the kinetic terms, we calculate second derivatives of $\mathcal{L}$ needed in Eq. (12). Using Eq. (3) we obtain

$$\mathcal{L}_{\psi_{A\mu}\psi_{B\nu}} = M^{AB} g^{\mu\nu} + N^{A\mu B\nu}. \tag{13}$$

The separation of the second derivative into the first and second terms in Eq. (13) is natural: $g^{\mu\nu}$ appears only in the first term, and N represents all remaining terms. N contains terms obtained by differentiating M and L with respect to $\psi_{B\nu}$, plus additional terms if ψ is a field strength. The ν index obtained from these $\psi_{B\nu}$ derivatives is attached to a field and not the metric $g^{\mu\nu}$. (Also, L does not contain an epsilon tensor since X does not.) Finally, notice that even though $\psi_{A\mu}$ and $\psi_{A;\mu}$ differ, the derivatives of $\mathcal{L}$ with respect to them coincide due to the form of the action (1). Thus the kinetic term becomes

$$\begin{aligned} \delta^2\mathcal{K} &= \left(\bar{M}^{AB} + \bar{N}^{A0B0}\right)\delta\bar{\psi}_{A;0}\delta\bar{\psi}_{B;0} \\ &+ \left(\bar{M}^{AB}\delta^{ij} - \bar{N}^{AiBj}\right)\delta\bar{\psi}_{A;i}\delta\bar{\psi}_{B;j}. \end{aligned} \tag{14}$$

We now prove that nonnegativeness of the kinetic term $\delta^2\mathcal{K}$ implies positive semidefiniteness of the matrix M. Indeed, suppose that M is not positive semidefinite. In such case, the matrix M has at least one negative eigenvalue, which means that there is a basis in which the matrix is diagonal with at least one negative entry, $\tilde{M} = \mathrm{diag}\,(\tilde{m}_1, \ldots, \tilde{m}_n)$ ($\tilde{m}_1 < 0$). (Quantities in this basis are designated with a tilde.) Let us choose such field variations that are nonzero only in the direction of the negative eigenvalue: $\delta\tilde{\psi}_{1;\mu} \neq 0$ and $\delta\tilde{\psi}_{A;\mu} = 0$ ($A > 1$). We further restrict $d-1$ quantities $\delta\tilde{\psi}_{1;i}$ to satisfy the following equation:

$$\tilde{N}^{1010}\delta\tilde{\psi}_{1;0}\delta\tilde{\psi}_{1;0} = \tilde{N}^{1i1j}\delta\tilde{\psi}_{1;i}\delta\tilde{\psi}_{1;j}. \tag{15}$$

These conditions make the kinetic term of Eq. (14) negative,

$$\delta^2\mathcal{K} = \tilde{m}_1\left[(\delta\tilde{\psi}_{1;0})^2 + \sum_i (\delta\tilde{\psi}_{1;i})^2\right] < 0, \tag{16}$$

thus proving that in order for $\delta^2\mathcal{K}$ to be nonnegative, the matrix M has to be positive semidefinite.

Using the result established in the previous paragraph, we conclude that solutions to the theory given by the action (1) are stable only if the matrix M is positive semidefinite.

Combining the relations between nonnegativeness of $\delta^2\mathcal{K}$ and positive semidefiniteness of M on one hand, and the NEC and positive semidefiniteness of M on the the other hand, we conclude that for the theory given by the action (1), only solutions satisfying the NEC can be stable.

We can deduce similar results for quantum systems. Suppose there exists a quantum state $|\alpha\rangle$ and a null vector n^μ such that

$$\langle\alpha|T_{\mu\nu}|\alpha\rangle n^\mu n^\nu = \langle\alpha|M^{AB}\Psi_A\Psi_B|\alpha\rangle < 0, \tag{17}$$

so that the NEC is violated in a quantum averaged sense. Define a basis $|\phi\rangle$ in which the operator $\mathcal{M} = M^{AB}\Psi_A\Psi_B$ is diagonal:

$$\mathcal{M}|\phi\rangle = m(\phi)|\phi\rangle. \tag{18}$$

Then violation of the quantum averaged NEC implies

$$\sum_{\phi\phi'}\langle\alpha|\phi\rangle\langle\phi|\mathcal{M}|\phi'\rangle\langle\phi'|\alpha\rangle = \sum_{\phi}|\langle\alpha|\phi\rangle|^2\, m(\phi) < 0. \tag{19}$$

This means that there exist eigenstates $|\phi\rangle$, whose overlap with $|\alpha\rangle$ is non-zero and on which the operator $\mathcal{M}$ has negative eigenvalues. This requires that M and hence $\delta^2\mathcal{K}$ is not positive semidefinite; by continuity, this must also be the case in a ball B in the Hilbert space of $|\phi\rangle$.

As a further consequence, we can conclude that a state $|\alpha\rangle$ in which the NEC is violated cannot be the ground state [8]. Suppose that $|\alpha\rangle$ is an energy eigenstate: $H|\alpha\rangle = E_\alpha|\alpha\rangle$. An elementary result from quantum mechanics is that $|\alpha\rangle$ can be the ground state only if

$$E_\alpha = \langle\alpha|H|\alpha\rangle \leq \langle\alpha'|H|\alpha'\rangle \tag{20}$$

for all normalized states $|\alpha'\rangle$ which need not be energy eigenstates. However, it is possible to reduce the expectation value of H by perturbing $|\alpha\rangle$. Specifically, we adjust $|\alpha\rangle$ only in the ball B, where we know from Eqs. (14)–(16) that there are perturbations which reduce the expectation of the kinetic energy without changing the expectation of the potential.

Note that the discussion above is in terms of unrenormalized (bare) quantities. The renormalized expectation $\langle\alpha|\mathcal{M}_{\mathrm{ren}}|\alpha\rangle = \langle\alpha|\mathcal{M}|\alpha\rangle - \langle 0|\mathcal{M}|0\rangle$ (where $|0\rangle$ is the flat-space QFT ground state) could be negative (e.g., as in the Casimir effect [1]), but this is possible only if $|\alpha\rangle$ is not $|0\rangle$.

In known cases of NEC violation, such as the Casimir vacuum or black hole spacetime, it is only the *renormalized* energy-momentum tensor which violates the NEC. As a simple example, consider a real scalar field ϕ. The energy-momentum tensor is simply $T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi$ plus terms proportional to $g_{\mu\nu}$ which do not play a role in the NEC. Then, $\mathcal{M} = (n^\mu\partial_\mu\phi)^2$ is a Hermitian operator with positive eigenvalues. Therefore, its expectation value in *any* state is positive: $\langle\alpha|\mathcal{M}|\alpha\rangle > 0$, for any $|\alpha\rangle$, including the Hartle-Hawking, Casimir or flat-space vacuum. We can verify this by direct calculation, computing the energy-momentum tensor using point-splitting regularization:

$$\langle 0|T_{\mu\nu}(x,x')|0\rangle n^\mu n^\nu = \frac{2[n^\mu(x-x')_\mu]^2}{\pi^2|x-x'|^6}, \tag{21}$$

which is manifestly positive. Note that $\langle\alpha|\mathcal{M}|\alpha\rangle > 0$ for all $|\alpha\rangle$, since the bare expectation is always dominated by the UV contribution. Now, had we taken a *negative* kinetic energy term for the scalar, the overall sign of $\mathcal{M}$ would change, allowing violation of the NEC. But, this model is clearly unstable, in accordance with our results.

C. Fermions

To this point we have only considered bosonic fields. We now extend our analysis to systems with fermions, adding to our Lagrangian the term

$$\mathcal{L}^{(\mathrm{f})} = \bar{\psi}(i\not{D} - m)\psi. \tag{22}$$

(A scalar-fermion coupling can be treated similarly, as can a Weyl fermion, whose determinant is the square root of the Dirac determinant.) Then, for any fixed gauge field background the fermions can be integrated out directly in favor of a non-local correction to the action for bosonic fields

$$-\sum_{\lambda_l > 0}\ln(\lambda_l^2 + m^2), \tag{23}$$

where $\not{D}\psi_l = \lambda_l \psi_l$ is the eigenvalue equation for the Dirac operator, with λ_l real. This shifts the energy-momentum tensor by

$$T^{(\mathrm{f})}_{\mu\nu} = -2|g|^{-\frac{1}{2}} \sum_{\lambda_l > 0} \frac{1}{\lambda_l^2 + m^2} \frac{\delta \lambda_l^2}{\delta g^{\mu\nu}}. \qquad (24)$$

Now write $\lambda_l^2 \psi_l = \not{D}^2 \psi_l = (g^{\mu\nu} D_\mu D_\nu - \frac{1}{2} i \sigma^{\mu\nu} F_{\mu\nu})\psi_l$ and use the orthonormality of the eigenfunctions to obtain

$$\lambda_l^2 = \int d^d x \, |g|^{\frac{1}{2}} \, g^{\mu\nu} \, \psi_l^\dagger D_\mu D_\nu \psi_l \ + \ \ldots, \qquad (25)$$

where the ellipsis denote terms which do not contain $g^{\mu\nu}$. After integration by parts, the contribution to the NEC from fermions is then

$$T^{(\mathrm{f})}_{\mu\nu} n^\mu n^\nu = \sum_{\lambda_l > 0} \frac{2}{\lambda_l^2 + m^2} \, (n \cdot D\psi_l)^\dagger (n \cdot D\psi_l). \qquad (26)$$

This additional contribution is always positive. So, the conclusions of the previous section are unmodified by the presence of fermions: violation of the NEC implies the bosonic kinetic energy is not positive semidefinite.

III. PERFECT FLUID

A macroscopic system may be approximately described as a perfect fluid if the mean free path of its components is small compared to the length scale of interest. For the dark energy, this length scale is of cosmological size. A perfect fluid is described by the energy-momentum tensor

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}, \qquad (27)$$

where ρ and p are the energy density and pressure of the fluid in its rest frame, and u_μ is its velocity. Let $j^\mu = J u^\mu$ be the conserved current vector (${j^\mu}_{;\mu} = 0$), and $J = (j_\mu j^\mu)^{\frac{1}{2}}$ the particle density.

The energy-momentum can be written [3, 7] as

$$T_{\mu\nu} = (f - Jf') g_{\mu\nu} + (f'/J) j_\mu j_\nu, \qquad (28)$$

where, comparing with Eq. (27), we have $\rho = f(J)$ and $p = Jf' - f$. The function $f(J)$ implicitly determines the equation of state.

The NEC for the tensor (28) becomes

$$T_{\mu\nu} n^\mu n^\nu = (f'/J)(j_\mu n^\mu)^2 \geq 0. \qquad (29)$$

Thus, perfect fluids with negative $f'(J)$ violate the NEC. Below, we demonstrate that $f'(J) < 0$ implies an instability.

Recall that the speed of sound in a fluid is given by $s = (dp/d\rho)^{\frac{1}{2}} = (Jf''/f')^{\frac{1}{2}}$, and that complex s implies an instability. Note $f'(J)$ cannot change its sign without producing an instability. Indeed, if it were to change its sign at some J_*, then s would be complex for either J larger or smaller than J_*, depending on the sign of $f''(J_*)$. (f'' cannot also change sign at J_*.) Therefore, if f' is negative anywhere, then it is negative everywhere, and to avoid complex s, f'' must be negative everywhere.

However, if f' and f'' are everywhere negative, then the fluid is unstable with respect to clumping. To see this, we first deduce the dependence of the fluid free energy F on particle number $N = JV$. Note that $(\partial F/\partial V)|_{T,N} = -p = N\partial(f/J)/\partial V$. By integration we find $F = N[f/J - h(T)]$, where the first term is just the energy E and $Nh(T) = TS$, where S is the entropy. It is easy to see that $(\partial^2 F/\partial J^2)|_{T,V} = V f''$.

Now consider two adjacent regions of the fluid with identical volumes. Suppose we transfer a small amount of matter δJ from one volume to another; the resulting change in total free energy is given by $\frac{1}{2} V f''(J)(\delta J)^2 < 0$. We see that the system can decrease its free energy by clumping into over- and under-dense regions. This itself is an instability, which results in a runaway to infinitely negative free energy unless the assumption of negative f' (violation of NEC) or negative f'' (real s) ceases to hold.

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